Turbo Joint Source-Channel Coding of Non-Uniform Memoryless Sources in the Bandwidth-Limited Regime

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Abstract—In this letter we investigate the one-layer coding/shaping system with single-level codes and sigma-mapping for the bandwidth-limited regime. Specifically, we consider non-uniform memoryless sources sent over AWGN channels. At the transmitter, binary data are encoded by a Turbo code composed by two identical RSC (Recursive Systematic Convolutional) encoders. The encoded bits are randomly interleaved and modulated before entering the sigma-mapper. The modulation employed in this system follows the unequal energy allocation scheme first introduced in [2]. At each time instant, the sigma-mapper superimposes multiple modulated symbols so as to output an amplitude signal. The receiver consists of an iterative demapping/decoding algorithm, which incorporates the a priori probabilities of the source symbols. Simulation results show that a significant improvement is obtained when the unequal energy allocation scheme is used rather than a standard BPSK modulation. Furthermore, the proposed scheme is compared with that proposed in [7] for the power-limited regime and unequal energy allocation.

Index Terms—Sigma-mapping, non-uniform memoryless sources, Turbo codes, bandwidth-limited regime.

I. INTRODUCTION

We consider the transmission of the information generated by a non-uniform memoryless source with probability distribution \(p_0, 1-p_0\) over the AWGN channel. Shannon’s Separation Theorem states that source coding and channel coding can be carried out in isolation. Thus, the standard approach has been to separate the encoding process in two parts: first, a source encoder capable of compressing the source up to its theoretical limit (given by its entropy \(H(p_0)\)), and second, a capacity-achieving channel code. Consequently, the \(E_{so}\) (average energy per source symbol) lower limit is given by

\[
\frac{E_{so}}{N_0} > \frac{2\rho - 1}{2R},
\]

where \(N_0\) is the one-sided noise power spectral density of the additive gaussian noise, \(R\) is the transmission rate (source symbols per channel symbol), and \(\rho/H(p_0) = 2R\) is the spectral efficiency in binary source symbols per two dimensions (2D). We will refer to joint source-channel coding schemes working at \(\rho/H(p_0) \geq 2\) as coding schemes operating in the bandwidth-limited regime; otherwise it is said that they are operating in the power-limited regime.

The above Separation Theorem was formulated in the context of potentially infinite-delay, lossless entropy-coding and infinite-blocklength channel coding. In practice, communication systems do not meet these ideal assumptions, and the overall performance can be improved if the tasks of source and channel coding are blended together by means of a joint source-channel encoder. In this way, the joint decoder can use some of the inherent redundancy of the source to alleviate the requirements of the channel encoder. For this reason, in the scheme proposed in this letter source and channel coding are jointly performed by a Turbo encoder operating at the bandwidth-limited regime in conjunction with a power-allocation strategy to achieve a shaping gain. To the best of our knowledge, previous work on joint source-channel coding for non-uniform memoryless sources has been done only for the power-limited regime. As to mention, in [7] they use LDPC codes, while [7], [8] employ non-systematic Turbo codes.

The remainder of the paper is organized as follows. In Section II we describe the proposed system, whereas Section III details the decoding process. Simulation results are presented in Section IV, and finally Section V concludes the paper.

II. PROPOSED SYSTEM

The proposed transmission system, which combines coding and shaping in an one-layer scheme, is depicted in Figure 1. The sequence \(\mathbf{u}\) of length \(K\), which is generated by a non-uniform memoryless binary source with a priori probabilities \(p_0\) and \(p_1\), is encoded through a Turbo code of rate \(R_c\), producing the encoded sequence \(\mathbf{c}\) of length \(N\) which is next processed through an interleaver \(\pi\). The interleaved version \(\tilde{\mathbf{c}}\) of \(\mathbf{c}\) is converted by a serial-to-parallel converter into \(k\) sequences \(\mathbf{v}^{(i)}\) of length \(L_i\), where \(0 \leq i \leq k-1\). Then, the modulator assigns different amplitudes (and consequently, different energies) to the encoded symbols, yielding \(k\) non-binary sequences \(\mathbf{s}^{(i)}\). Following the scheme in [7], the symbols associated to systematic bits 1 and 0 are, respectively,

\[
\sqrt{E_1} = \sqrt{p_0/p_1} \quad \text{and} \quad -\sqrt{E_0} = \sqrt{p_1/p_0},
\]

where \(p_1 E_1^0 + p_0 E_0^0 = 1\). For the parity bits, the associated symbols depend on the value of both the parity and the input bit of the underlying RSC (Recursive Systematic Convolutional) code of

\[\text{Notice that } R_c \text{ (code rate) is not necessary equal to } R \text{ (transmission rate) defined in expression (1).}\]
the Turbo encoder. Notice that the input sequence of one RSC code corresponds to the systematic bits, while the input of the other one corresponds to the interleaved version of the source sequence. The amplitudes of parity bits 1 and 0 are given by $+\sqrt{E_p}$ and $-\sqrt{E_p}$, respectively, where the subindex $i$ will take the value $0$ or $1$ depending on the value of the associated input bit. $E_p$ is set to $(1 - \theta)/p_1$, and $E_p^0$ to $\theta/p_0$, where $p_1E_p^0 + p_0E_p^0 = 1$. The value of $\theta$ ($0 \leq \theta \leq 1$) is chosen by simulation so as to minimize the probability of error at the receiver, which is usually achieved when $\theta = 0.5$ [2]. Thus, systematic bits can be mapped to 2 different values, while parity bits to 4 different values.

Fig. 1. Proposed Turbo encoding, modulation and sigma-mapping scheme.

These modulated sequences enter the sigma-mapper $\Sigma$ [2], which generates a single signal sequence $x$ of length $L$. At time $t$, the output of the sigma-mapper is denoted as $x_t = \sigma_2(s_t)$, where $s_t = (s_t(0), \ldots, s_t(k-1))$ and $\sigma_2(s_t) \triangleq \sum_{i=0}^{k-1} \alpha_i s_t(i)$. The sigma-mapper utilized in our scheme is known as type-I, since $\alpha_i = \alpha \forall i \in \{0, \ldots, k-1\}$. The value of $\alpha$ is chosen to satisfy $E_{x_t} = E_x$ and it can be shown that $\alpha$ should be set to $\sqrt{E_x/E_f}$. The received sequence $y$ at destination is a version of $x$ corrupted by Additive White Gaussian Noise (AWGN).

III. DECODING PROCESS

The receiver shown in Figure 2 iterates between the sigma-demapper (labeled as $\Sigma^{-1}$) and the Turbo decoder. The sigma-demapper is based on a SISO (Soft-Input Soft-Output) demapping algorithm, and the Turbo decoder is implemented by the Sum-Product Algorithm (SPA) applied to the Factor Graph (FG) that describes the Turbo code. The decoding procedure allows for the successive exchange of extrinsic probabilities between the sigma-demapper and the Turbo decoder, which iteratively refines the a posteriori probabilities of the original source symbols. The sigma-demapper processes the sequence $y$ and estimates the probabilities of the $k$ superimposed symbols contained in each channel symbol $y_t$ ($1 \leq t \leq L$). If the symbol corresponds to a systematic bit, the sigma-demapper generates the extrinsic probability $P_v^{(e)}(m)$, where $m \in \{0, 1\}$. Accordingly, if the symbol represents a parity bit, the demapper calculates the probabilities $P_v^{(e)}(m|\text{input bit})$, where the input bit can take values 0 and 1.

Hereafter, the superscript (initial) represents the non-uniformity of the source. Thus, for the systematic bits, the probabilities $P_v^{(\text{initial})}(m)$ will be $p_0$ for $m = 0$ and $p_1$ for $m = 1$, while for the parity bits they are always set to 0.5. The superscript (a) refers to the a priori probabilities coming from the Turbo decoder. In the first iteration they are equal to 0.5. Although the initial probabilities of the source can be estimated by both the encoder and the decoder [2], we will assume they are known in order to achieve optimum performance.

For all $V^{(i)}$ corresponding to systematic bits, the extrinsic probability $P_{v^{(i)}}^{(e)}(m)$ generated by the sigma-demapper is proportional to

$$P_{v^{(i)}}^{(e)}(m) \propto P_v^{(\text{initial})}(m) \prod_{y | P_y} \mathbb{I}(y = m) P_{y|V}(y|\phi(s))$$

where $\mathbb{I}(P)$ denotes the k-extension of the binary Galois Field, and $\mathbb{I}(P)$ is the indicator function which takes the value 1 when the proposition $P$ is true and 0 otherwise. In the above expression, $P_{v^{(i)}}^{(e)}$ is given by

$$P_{v^{(i)}}^{(e)} = \prod_{j \neq i} P_v^{(\text{initial})}(m_j) P_v^{(a)}(m_j),$$

whereas $P_{y|V}(y|\phi(s))$ is proportional to

$$P_{y|V}(y|\phi(s)) \propto \sum \exp \left( -\frac{(y - \phi(s))^2}{2\sigma^2} \right),$$

where the number of elements in the sum varies depending on the number of parity and systematic bits that compose the vector $v$. The reason being that for a given value of a systematic bit, there is only one possible value of its associated modulated symbol $s$ while for a parity bit, two values of $s$ are possible depending on the value of the associated input bit. For example, consider the case where $k = 2$ and $(V^{(1)}, V^{(2)}) = (0, 0)$. If $V^{(1)}$ and $V^{(2)}$ are both systematic, the value of $\phi(s)$ is unique and equal to $-2\sqrt{p_1/p_0}$. On the contrary, if one is systematic and the other a parity bit, two values of $\phi(s)$ are possible, namely, $-\sqrt{p_1/p_0} - \sqrt{1 - \theta}/p_0$ and $-\sqrt{p_1/p_0} - \sqrt{1 - \theta}/p_0$. Finally, when $V^{(1)}$ and $V^{(2)}$ are both parity bits, $\phi(s)$ may take four different values. Besides, when the vector $v$ contains parity bits, the exponential term of the righthand side of expression (4) has to be multiplied by the a priori probabilities of the input bits associated with such parity bits. When calculating the extrinsic probabilities $P_{v^{(e)}(m|\text{input bit})}$ of the parity bits, the only difference is that in expression (4) we do not have to multiply the exponential term by the a priori probability of the input bit associated with the parity bit we are considering. Furthermore notice that expression (2) is not multiplied by $P_{v^{(0)}}^{(e)}(m = m)$ to avoid any positive feedback to the Turbo decoder.

Fig. 2. Decoding scheme.

Once computed by the sigma-demapper, all the above extrinsic probabilities are passed to the Turbo decoder through a parallel-to-serial converter and the deinterleaver $\pi^{-1}$. The Turbo decoder employs these probabilities as a priori probabilities, and runs the SPA algorithm over the FG that describes its compounding RSC codes. However, note that a slight modification is required in such FG, since in our scheme extrinsic information for the parity bits is also needed to be passed to the sigma-demapper. The extrinsic probabilities generated by the Turbo decoder are then used as a priori probabilities by the sigma-demapper. The decoder stops after a fixed number of iterations.
IV. Simulation Results

In order to verify the performance of the proposed scheme, two sets of simulations have been performed. The goal of the first set is to assess the improvement that the unequal energy allocation strategy entails in the overall performance of the proposed scheme. To that end, the proposed system is simulated with and without energy allocation. In the latter case the encoded symbols at the output of the Turbo coder are simply BPSK modulated (i.e. by using equal energy). The Turbo code from [?] Example A with rate $R_c = 1/3$ and generator polynomial $G(D) = 1/(1+D)$ has been used. The value of $k$ was set to $k = 3$, so the transmission rate $R$ is 1 binary source symbol per dimension, i.e. a spectral efficiency of 2 (bandwidth-limited regime).

Monte Carlo simulations have been performed for a block-length $K = 10000$ and a maximum of 50 decoding iterations. Three source symbol distributions $p_0 \in \{0.1, 0.2, 0.3\}$ have been selected, giving rise to source entropies $H(p_0) = 0.47, 0.72$ and 0.88 bits per source symbol, respectively. Figure 3 shows the BER versus $E_{so}/N_0$, where the vertical lines correspond to the theoretical limit given by expression (1). Notice that for $p_0 = 0.1$, the performance improvement when utilizing, in our scheme, an unequal energy allocation modulation instead of standard BPSK is 2.37 dB at a BER = $10^{-4}$. On the other hand, observe that the proposed scheme is $2.84$ dB away from the Shannon limit. Further simulations included in the plot show that, for $p_0 = 0.2$ and $p_0 = 0.3$, we obtain a gain of 1.1 and 0.37 dB, respectively, and they are 2.08 and 1.66 dB away from their theoretical limit.

The above gaps to the Shannon limit can be reduced by using a more powerful Turbo code. This is shown in the the second set of simulations, where the goal is also to compare our scheme operating at the bandwidth-limited regime with the Turbo coding scheme proposed in [?] (suitable for operating only at the power-limited regime). In this comparison, both systems use the same Turbo code of rate $R_c = 1/3$, generator polynomial $G(D) = (1+D+D^2+D^3)/(1+D^3+D^4)$ and $K = 16384$. Notice that although both schemes utilize the same Turbo code of rate $R_c = 1/3$, our scheme implements a spectral efficiency of 2 ($R = 1$), whereas the scheme in [?] one of 2/3 ($R = 1/3$). Figure 4 plots the BER versus $E_{so}/N_0$ for $p_0 = 0.005$, where the vertical lines represent the corresponding theoretical limits of both systems. Although both schemes need approximately the same $E_{so}/N_0$ to obtain a BER of $10^{-5}$, our system performs slightly better, since its corresponding Shannon limit is closer. Further results show that for $p_0 = 0.01$, both schemes are 2.0 dB away of their corresponding theoretical limits.

Fig. 3. BER versus $E_{so}/N_0$ for $p_0 \in \{0.1, 0.2, 0.3\}$.

Fig. 4. BER versus $E_{so}/N_0$ for $p_0 = 0.005$.

V. Conclusion

We have proposed a one-layer coding/shaping system with single-level codes and sigma-mapping, for the case of non-uniform memoryless sources sent over AWGN channels. The proposed system has proved to have similar error rate performance, in the bandwidth-limited regime, to the system proposed in [?] for the power-limited regime. This satisfactory behavior of the BER versus $E_{so}/N_0$ is mostly due to the unequal energy allocation strategy implemented.

REFERENCES


