Indoor localization with range-based measurements
and little prior information

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Abstract—We address the problem of indoor localization with networks of sensors taking range-based measurements in presence of very little prior information. We propose several robust methods that do not require previous measurement campaigns when a network is deployed. The focus is on networks of ultrawideband sensors, but the proposed range-based methods can be applied to other types of sensor networks. The location of a target node is estimated from measured distances to anchor nodes of known positions. We allow for the possibility of large errors in the range measurements due to Undetected Direct Path (UDP) propagation conditions. In mitigating the UDP effect, our approach is to combine intermediate location estimates from different subsets of beacons. We propose novel criteria for identifying the combinations that produce bad estimates. These combinations are then discarded in obtaining the final estimation. Results with the proposed methods achieve improved performance with respect to that of existing techniques that exploit the same prior information. Under many scenarios, the proposed methods reach the performance of some algorithms that exploit more prior information.

I. INTRODUCTION

Accurate localization of nodes of a sensor network is key for a wide variety of applications including inventory tracking, security, monitoring, and navigation aids [1], [2]. In indoor environments where the Global Positioning System does not have good performance, one relies on wireless localization, where the location estimates of the targets are usually estimated from power, time of arrival (TOA), time difference of arrival, or angle of arrival of radio signals exchanged among the nodes in the network [3].

In this paper we focus on indoor localization with Ultra Wideband (UWB) signals. The UWB technology has emerged as a promising alternative [4], [5] because of both, its accurate ranging and obstacle penetration capabilities and the potential for low power and low cost systems. More specifically, it allows for the acquisition of accurate range estimates by means of TOA measurements for as long as rays propagating along direct paths (DPs) between beacons (anchors) of known locations and targets of unknown locations (but known to lie within a specific service area) can be detected [6].

Range-based approaches are the most popular with UWB since measurements from three detected direct paths (DDPs), lead to a reliable location estimate. However, the DP may be obstructed or attenuated below the detection threshold, a situation referred to as undetected direct path (UDP). The latter implies that another ray corresponding to a trajectory longer than the DP is erroneously regarded as the direct ray, leading to large estimation errors [7], [8].

In general, when the number of DDP measurements is sufficient for accurate localization of a target, a presence of UDP measurements can strongly degrade the positioning accuracy. This has led to many efforts directed at mitigating the UDP effect. As a result, various methods have been proposed, relying on different prior information. This information can be of statistical nature, such as probability density functions (PDFs) of the range errors under DDP/UDP conditions, or prior probabilities of DDP/UDP occurrence [9]; partial knowledge of the topology of the service area [10] or fingerprinting databases [11], [12].

UDP mitigation strategies may be grouped according to the amount of prior information they use. Clearly, a better performance is expected from algorithms that exploit more information, but which may be difficult to obtain and may need maintenance in case of environmental changes. To avoid these problems, in this paper we propose methods that exploit as little prior information as possible. In particular, these methods do not use prior probabilities of DDP/UDP, topology of the service area, and processing of received waveforms (for example, to identify the propagation conditions of specific beacons, as in [13], [14]).

We first present a method that does not require a range error model or statistics. We then describe a method that uses knowledge of the range error model of DDP only. For constant bandwidths, the errors of a DDP model are caused by multipath, and therefore, the model is relatively accurate for different environments, whereas a UDP model is often environment specific and depends on the size and number of major metallic objects in the environment [15].

The rest of the paper proceeds as follows. In Section II, a range error model is described. In Section III, the novel localization methods are proposed. Section IV contains brief descriptions of the methods that are used for comparisons, whereas Section V presents and comments the results of the simulations. Finally, Section VI provides some conclusions.

II. MODEL FOR RANGE MEASUREMENTS AND NOTATION

A variety of efforts to model range errors in wireless networks based on experimental work has been reported in
the literature [16]. In generating indoor UWB range measurements, we use the model from [17]. The reason for this choice is that this model clearly distinguishes the DDP and UDP cases. It should be emphasized that the DDP and UDP concepts are not identical to situations referred to in the literature as line of sight (LOS) and non-LOS (NLOS). LOS pertains to the presence of a physical line of sight, while detected direct path (DP) relates to the detectability of the first cluster of paths above the threshold, leading to a large range error. For UWB signals, the UDP conditions have been observed to be caused mainly by large metallic objects [15]. According to [17], $\epsilon_{n,UDP}$ is also Gaussian but with larger mean and variance than that of $\epsilon_{n,DDP}$. In particular [17],

$$m_{n,UDP} = m_\gamma \log(1 + d_n) + m_U$$

(6)

and

$$\sigma^2_{n,UDP} = \sigma^2_\gamma [\log(1 + d_n)]^2 + \sigma^2_U.$$  

(7)

In the sequel we consider the numerical values for a bandwidth of 500 MHz, $m_U = 1.62$ m and $\sigma_U = 0.809$ m [17].

In summary, the range measurement $r_n$ is Gaussian, i.e., $r_n \sim \mathcal{N}(d_n + m_{n,DDP}, \sigma^2_{n,DDP})$ if the measurement is of a DDP, or $r_n \sim \mathcal{N}(d_n + m_{n,UDP}, \sigma^2_{n,UDP})$ if it is of a UDP.

III. PROPOSED LOCALIZATION METHODS

In this section, we propose several localization methods with a design constraint of relying on as little prior information as possible. The methods use subsets of measurements of three or more beacons (the maximum number of subsets is $K = \sum_{i=3}^N \binom{N}{i}$). Each combination is identified by an index $k$. We denote by $\mathcal{I}_k$ the subset of beacon indices included in the $k$th combination, while $\mathcal{I}_k^c$ is the complementary subset of indices. In methods that do not use range error models for the UDP case, it is preferable to discard the UDP measurements from the computation of the location estimate [7]. The main problem is that it is not known which measurements are UDP measurements, and therefore, combinations that include them have to be identified and removed from the computation of the location estimate.

We first propose a localization method that does not use any range error model and subsequently, we describe two variations of that method which have a reduced complexity. Then we allow for the use of a range error model, but only for the DDP case, and propose three other localization methods.

A. Proposed method with No Error Model Information (NEMI)

We first propose a method that shares the structure of the Rwgh method from [18]: for each possible combination of three or more beacons, an intermediate location estimate is considered, and the final estimate is obtained by combining the intermediate estimates. Since no error model information is available, the intermediate estimates are determined through the classical Least Squares (LS) technique. There are two important differences of our method with respect to Rwgh. The first is that it discards some of the beacon combinations in computing the final estimate, and the second is in the criterion for deciding whether to discard a certain combination of beacons. Namely, we propose a novel criterion that uses beacon measurements that are not part of the combination.

Consider the $k$th subset of beacons. Let $\hat{z}_k = (\hat{x}_k, \hat{y}_k)$ be the corresponding intermediate location estimate. We can compare the $n$th range measurement $r_n$ with the estimated distance between the $n$th beacon and the target, and the estimated error is

$$\hat{\epsilon}_{n,k} = r_n - \hat{d}_{n,k}$$  

(8)
where
\[ \hat{d}_{n,k} = \| \hat{z}_k - z_n \| = \sqrt{(\hat{x}_k - x_n)^2 + (\hat{y}_k - y_n)^2}. \] (9)

If all the measurements were accurate, the intermediate location estimate \( \hat{z}_k \) obtained from some of them would be a good one, and the range measurements \( r_n \) would be close to the estimated distances \( \hat{d}_{n,k} \). This implies that the estimated errors \( \hat{\varepsilon}_{n,k} \) would have relatively small absolute values.

This idea is also followed in [18]. However, our novelty can be explained as follows. Suppose first that \( \hat{z}_k \) is obtained from DDP measurements, which entails that it is a good estimate. Consider now the remaining measurements, which were not used in obtaining \( \hat{z}_k \). If one of these measurements is a DDP measurement, its value will be close to the estimated distance computed according to (9). If, instead, it is a UDP measurement, its error \( \hat{\varepsilon}_{n,k} \) is large and positive. We exploit this in the proposed method. Namely, when the examined combination is good, the errors of the measurements left out of the combination are either small or are large with positive values. If, on the other hand, the \( k \)th combination contains a beacon with one or more UDP measurements, there may be left out measurements with a large negative value of \( \hat{\varepsilon}_{n,k} \). This kind of measurement is inconsistent with the hypothesis that \( \hat{z}_k \) is an accurate estimate, and therefore we discard the \( k \)th combination from the computation of the final location estimate.

Our reasoning is illustrated by Figure 1. Figure 1(a) shows the locations of four beacons and a target. Beacons 1, 2 and 3 are in DDP connections with the target, while beacon 4 is under a UDP propagation condition (though the obstacles are not shown in the figure). The intermediate estimate corresponding to the \( k \)th combination (beacons 1, 2 and 3), \( \hat{z}_k \), is also shown. Since the three beacons have DDP measurements, it is a good location estimate. The estimated error for the fourth range measurement, \( \hat{\varepsilon}_{4,k} = r_4 - \hat{d}_{4,k} \), is positive with large value, suggesting a UDP measurement consistent with the intermediate estimate. In Figure 1(b), the combination \( \mathcal{J}_{k+1} = \{2, 3, 4\} \) contains a beacon with a UDP measurement (beacon 4). Due to \( r_4 \), the intermediate estimate of this combination is poor. The error of the left out beacon (beacon 1), \( \hat{\varepsilon}_{1,k+1} = r_1 - \hat{d}_{1,k+1} \), is negative with a large absolute value, revealing an inconsistency and suggesting that the intermediate estimate \( \hat{z}_{k+1} \) should be discarded from the final location estimate.

Based on the above reasoning, we propose a localization method which we refer to as No Error Model Information (NEMI). In deciding whether \( \hat{\varepsilon}_{n,k} \ll 0 \), we use a threshold \( \tau_1 \). The effect of its value is studied in Section V.

Note that the full set of beacons does not have a complement and therefore cannot be tested. If all the subsets pass the test, the full set is also accepted; otherwise, it is discarded. In the case that there are no good subsets, one can use the estimate of the full set with the warning that it is probably obtained by using one or more UDP measurements.

**B. Proposed NEMI Methods with Reduced Complexity (NEMI Methods)**

The NEMI and Rwh methods need to compute the intermediate estimates from all the possible combinations of measurements. Therefore, it is important to consider a reduction in complexity of NEMI by avoiding the computation of some of the intermediate estimates.

To this end we propose a variation of the NEMI method, which we refer to as the No Error Model Information and Reduced Complexity I (NEMIRC I) method. According to the method, the combinations of three or more beacons are considered in increasing order of number of beacons, and the NEMI method is applied. If a certain combination does not meet the criterion for inclusion in the final estimate, the combination most likely contains one or more UDP measurements. Therefore, combinations of measurements that have this combination as a subset are a priori discarded. For example, if there are five beacons and the combination of the beacons 1-2-3 is discarded, we automatically discard the combinations 1-2-4, 1-2-5, 1-3-4, 1-3-5, 1-4-5, 2-3-4, 2-3-5, 2-4-5, 3-4-5.
NEMI algorithm

\[
\text{for } k = 1 : K - 1, \text{ do} \\
\hat{\alpha}_k &= \arg\min_z \left\{ \sum_{n \in \mathcal{I}_n} [r_n - \|z - z_n\|^2] \right\} \\
\alpha_k &= 1 \\
\text{for each } n \in \mathcal{I}_k, \text{ do} \\
\hat{\epsilon}_{n,k} &= r_n - \|\hat{z}_k - z_n\| \\
\text{if } \hat{\epsilon}_{n,k} < \tau_1 \text{ then } \alpha_k = 0 \\
\text{end if} \\
\text{end for} \\
\text{if } \left( \prod_{k=1}^{K-1} \alpha_k = 1 \right) \text{ or } \left( \sum_{k=1}^{K-1} \alpha_k = 0 \right) \text{ then} \\
\hat{z}_K &= \arg\min_z \left\{ \sum_{n \in \mathcal{I}_K} [r_n - \|z - z_n\|^2] \right\} \\
\alpha_K &= 1 \\
\text{else } \alpha_K = 0 \\
\text{end if} \\
\hat{z} &= \frac{\sum_{k=1}^{K} \alpha_k \hat{z}_k}{\sum_{k=1}^{K} \alpha_k}
\]

1-2-3-4 and 1-2-3-5 without determining the corresponding intermediate estimates.

An alternative to NEMIRC I is NEMIRC II, which has even less computations. According to NEMIRC II, the objective is to find as early as possible a set of three nodes that meets the criterion for contribution to the final estimate. Once such a set is determined, all its nodes are classified as DDP nodes. Then we use the obtained set to classify the remaining nodes. We do it by replacing one of the three nodes with one of the remaining nodes. If the new combination meets the criterion, the new node is classified as a node with a DDP measurement; otherwise it is discarded. We proceed in the same way until all the nodes are tested. Upon the completion of this step, we have the set of nodes that are considered to be with DDP measurements. We then apply the NEMIRC I method on these nodes.

We now derive the lower bound for the number of combinations for which an intermediate estimate is computed for the NEMIRC I method. Note that the first step is to try all the combinations of three beacons. If we discard all but one, this is the case of minimum number of computed estimates, because all the rest of combinations of four or more beacons will automatically be discarded. Thus, the lower bound is the number of combinations of three beacons, \( \binom{N}{3} \).

Next we find the minimum number of intermediate estimates of NEMIRC II. Recall that the first step of the method is the examination of combinations with three beacons. The minimum number is obtained when we discard all of the combinations except the first one that is examined. Suppose that the first combination of nodes, say 1, 2, and 3, passes the test. Let the next examined combination be composed of nodes 1, 2, and 4, and suppose that it is not accepted. If one hypothesizes that the rejection of the combination is because of node 4, and that it is due to its UDP measurement, then all the combinations with node 4 are immediately discarded. Let the subsequent combination be 1, 2, and 5, and again if this combination is rejected, it is node 5 that is considered with a UDP measurement. Therefore all the combinations with node 5 are also discarded. If we continue in this manner, and if each of the new nodes combined with nodes 1 and 2 does not provide a good combination, the final estimate is obtained only from the combination of nodes 1, 2, and 3. In this case, according to NEMIRC II, the minimum number of intermediate estimates is computed, and it is equal to \( N - 2 \). This result, however, is obtained in a very specific scenario when (a) exactly three beacons are with DDP measurements and (b) these beacons are identified in the first tested combination.

For a given value of \( N \) and a given probability of UDP condition, \( P(\text{UDP}) \), we derive a benchmark for the mean number of tested combinations as

\[
B(N, P(\text{UDP})) = \sum_{i=0}^{N} b(N,i) P(N_{\text{DDP}} = i)
\]

where \( N_{\text{DDP}} \) is the true number of DDP beacons and \( b(N,i) \) is a lower bound of the number of tested combinations conditioned on correct decisions and for \( N_{\text{DDP}} = i \). When \( 3 \leq N_{\text{DDP}} \leq N \), if the first combination of three nodes is accepted, the number of tested combinations reaches the bound, whose value is the sum of the total number of possible combinations of DDP beacons plus one combination for each of the \( N - N_{\text{DDP}} \) UDP beacons:

\[
b(N, N_{\text{DDP}}) = \sum_{i=3}^{N_{\text{DDP}}} \binom{N_{\text{DDP}}}{i} + N - N_{\text{DDP}}.
\]

In case of less than three \( N_{\text{DDP}} \) beacons the algorithm will test all the possible combinations \( b(N, N_{\text{DDP}}) = \sum_{i=3}^{N} \binom{N}{i} N_{\text{DDP}} \leq 2 \), and we can express the benchmark as

\[
B(N, P(\text{UDP})) = \sum_{i=3}^{N} \binom{N}{i} \sum_{i=0}^{2} P(N_{\text{DDP}} = i) + \sum_{i=3}^{N} \left( \sum_{l=0}^{i} \binom{i}{l} + N - i \right) P(N_{\text{DDP}} = i)
\]

C. Proposed method based on a model with Partial Range Error Information (PREMI)

We now propose a method that exploits partial knowledge of the range error. This knowledge is summarized by a probabilistic model of the error in the DDP case. Since the range error in this case is caused by multipath and depends on the bandwidth, the validity of its statistics can be expected to hold for different indoor environments. By contrast, in the UDP scenario, the error statistics are dependent on the specific indoor environment (in particular on the number and position of large metallic objects in the service area [15]). We chose not to use a model for the UDP measurements so that there is no need for measurement campaigns.

We refer to the proposed method as Partial Range Error Model Information (PREMI) method, which shares the struc-
ture of the NEMI method. The $K$ combinations are considered and used to compute intermediate location estimates under the assumption that all the beacons are in DDP connection with the target. One difference is the replacement of the LS estimates by Maximum Likelihood (ML) estimates. This is possible because the likelihood function of the range measurements in the DDP case is known.

Once an intermediate ML estimate $\hat{z}_k$ is available, the error $\varepsilon_{n,k}$ of a DDP measurement is modeled as a Gaussian RV with a mean

$$m_{n,DDP} = m_\gamma \log(1 + \hat{d}_{n,k})$$

and a standard deviation

$$\sigma_{n,DDP} = \sigma_\gamma \log(1 + \hat{d}_{n,k}).$$

The combination is tested by using a criterion derived from this model. More specifically, as in the NEMI algorithm, we compare $\hat{\varepsilon}_{n,k}$ with a threshold, where the threshold, $\tau_{2,n,k}$ is obtained from

$$P(\varepsilon_{n,k} \leq \tau_{2,n,k}) = \zeta, \text{ for } n \in I_k$$

and the assumption that $\varepsilon_{n,k}$ is Gaussian with mean and variance given by (12) and (13), respectively, and obtained from the measurements of the $k$th combination, whereas $\zeta$ is a predefined probability of false alarm.

In completing the description of this method, we state the likelihood function that is used in computing the ML estimate. Let $r_k$ be the vector containing the measurements included in the $k$th combination, that is, $\{r_n\}, n \in I_k$. With the assumption that the measurements are statistically independent, the PDF of $r_k$ reads

$$p(r_k; z) = \prod_{n \in I_k} p_{DDP}(r_n; z)$$

where $p_{DDP}(r_n; z)$ is the PDF of $r_n$ parameterized by $z$. We can readily show that the log-likelihood function is given by (within an irrelevant additive constant independent of $z$)

$$\Lambda(r_k; z) = -\sum_{n \in I_k} \ln \sigma_{n,DDP} - \frac{\sum_{n \in I_k} (r_n - d_n - m_{n,DDP})^2}{2\sigma_{n,DDP}^2}$$

The ML estimator maximizes $\Lambda(r_k; z)$ over all the possible target locations $z$. Note that the maximization must be performed for $z$ varying over the floor plan.

D. Proposed PREMI methods with Reduced Complexity (PREMIRC)

We also propose methods with Partial Range Error Model Information and Reduced Complexity (referred to as PREMI I and PREMIRC II). They are similar to the PREMI method but avoid the computation of intermediate ML estimates in analogous ways as do the NEMIRC I and NEMIRC II methods when compared to the NEMI method.

IV. OTHER LOCALIZATION TECHNIQUES FOR COMPARISON

In this section, we briefly overview some existing localization methods. For a meaningful comparison and for ease of reading of the simulations section, we divide the algorithms into two categories: methods without error model knowledge, and methods with range error model knowledge in the DDP case.

A. Methods without range error model knowledge

1) The classical LS technique: The location estimate is computed by minimizing the sum of squared residues

$$\hat{z} = \arg \min_z \left\{ \sum_{n=1}^N [r_n - \|z - z_n\|^2] \right\}.$$  

2) The Rwgh method [18]: For each of the $K$ possible subsets of beacons, an intermediate estimate is obtained by

$$\hat{z}_k = \arg \min_z \left\{ \sum_{n \in I_k} [r_n - \|z - z_n\|^2] \right\}$$

followed by computing a normalized quantity for each $\hat{z}_k$ according to

$$\Upsilon(\hat{z}_k; I_k) \triangleq \frac{\sum_{n \in I_k} [r_n - \|\hat{z}_k - z_n\|^2]}{\text{Size of } I_k}.$$

PREMI algorithm

for $k = 1 : K - 1$, do

$$\hat{z}_k = \arg \max_z \left\{ -\sum_{n \in I_k} \ln \sigma_{n,DDP} - \frac{\sum_{n \in I_k} (r_n - d_n - m_{n,DDP})^2}{2\sigma_{n,DDP}^2} \right\}$$

$$a_k = 1$$

for each $n \in I_k$, do

$$\hat{d}_{n,k} = ||\hat{z}_k - z_n||$$

$$\hat{\varepsilon}_{n,k} = r_n - \hat{d}_{n,k}$$

Obtain $\tau_{2,n,k}$ from $P(\varepsilon_{n,k} \leq \tau_{2,n,k}) = \zeta$

if $\hat{\varepsilon}_{n,k} < \tau_{2,n,k}$, then $a_k = 0$

end if

end for

end for

if $\left( \prod_{k=1}^{K-1} a_k = 1 \right)$ or $\left( \sum_{k=1}^{K-1} a_k = 0 \right)$ then

$$\hat{z}_K = \arg \max_z \left\{ -\sum_{n \in I_K} \ln \sigma_{n,DDP} - \frac{\sum_{n \in I_K} (r_n - d_n - m_{n,DDP})^2}{2\sigma_{n,DDP}^2} \right\}$$

$$a_K = 1$$

else $a_K = 0$

end if

$$\hat{z} = \frac{\sum_{k=1}^{K} a_k \hat{z}_k}{\sum_{k=1}^{K} a_k}$$
The final estimate of $z$ is the weighted combination,

$$
\hat{z} = \frac{\sum_{k=1}^{K} \hat{z}_k (Y(\hat{z}_k; I_k))^{-1}}{\sum_{k=1}^{K} (Y(\hat{z}_k; I_k))^{-1}}.
$$

(20)

It should be pointed out that in [18] the possibility of using less combinations in the final estimate is considered too. The combinations are ordered according to the residues of their intermediate estimate, and only a subset of the combinations is picked for the final estimate, always starting from the ones with minimum residues. However, this approach does not result in any significant complexity saving, because the intermediate estimates need to be computed anyway for all the combinations in order to find out the residues. Besides, in [18] it is shown that these variations of the original algorithm do not provide any significant improvement in performance.

3) The LS method with knowledge of the DDP/UDP configuration: We now consider a method that has perfect knowledge of the DDP/UDP status of each beacon with respect to the location of the target. The algorithm takes only the subset of beacons in DDP connection with the target (denoted by $I_{DDP}$) and applies the classical LS technique, i.e.,

$$
\hat{z} = \arg\min \sum_{n \in I_{DDP}} \left\{ \sum_{r \in \{\text{DDP}, \text{UDP}\}} \left[ r_n - \|z - z_n\|^2 \right] \right\}.
$$

(21)

Even though it is unrealistic that we have this knowledge, we include it here for benchmark purposes, as it was done in [18]. When $N_{DDP} < 3$, the method takes the whole set of beacons.

B. Methods with DDP range error model knowledge

1) The naïve ML method: This is the most naïve among the possible approaches with DDP error model knowledge. It applies the ML approach to all the beacon measurements as if they are all in DDP connection with the target. The ML estimate is obtained by minimizing (16), where $I_k$ is replaced by the complete set of beacons.

2) The method from ref. [9]: This method uses range error models for both the DDP and UDP cases. It also assumes the prior probabilities of DDP/UDP ($P_n^{(DDP)}$ and $P_n^{(UDP)}$) for each beacon and treats them as being independent of the target location (which altogether is unrealistic). The position estimate is obtained by maximizing the function

$$
p(r; z) = \prod_{n=1}^{N} \left[ p_{DDP}(r_n; z) \cdot P_n^{(DDP)} + p_{UDP}(r_n; z) \cdot P_n^{(UDP)} \right].
$$

(22)

Since the method exploits more information than the proposed ones, it can be thought of as a benchmark.

3) The ML method with knowledge of DDP/UDP configuration: This method assumes the perfect knowledge of which beacons are DDP and which UDP, and therefore the ML approach is applied only to the beacons in DDP condition, unless $N_{DDP} < 3$, in which case the method boils down to the naïve ML method. Since the method is based on an unrealistic assumption, it is also used for benchmark purposes.

V. SIMULATION RESULTS

In our simulations, we considered a rectangular area of 10m × 5m. The beacons were located at its borders, as is typical in the wide literature. There was a total of 2000 random target locations, and for each location we generated 10 different realizations of range measurements. For each simulation trial, the DDP/UDP status of each beacon was randomly determined according to prescribed values of the probability of UDP condition, $P^{(UDP)}$.

Letting $\hat{z}$ be the estimate of the target position provided by a given algorithm, the estimation performance was expressed by the root mean square error (RMSE), which is defined as the square root of the average of $\|z - \hat{z}\|^2$ taken over $z$. The number of beacons $N$ varied from 3 to 8. The first four were placed in the corners, the 5th and 6th were at the mid points of the longer walls, while the 7th and 8th lied at the mid points of the shorter walls. Although the rectangular floor plan is kept throughout the simulations, the variety of simulations with respect to the number of beacons and probabilities of UDP condition reflects numerous scenarios with different densities of beacons and of objects capable of causing UDP conditions seen in practice. All the algorithms searched for the target within the perimeter of the service area. The conclusions of the study are not expected to change much with the shape of the service area as long as the distribution of the beacons along the border of the service area is balanced.

A. Methods without knowledge of range error model

Figure 2 shows the RMSE for the different methods that do not use any range error model. The number of beacons was $N = 6$, and $\tau_1 = -0.5$ m. The RMSE is shown as a function of $P^{(UDP)}$. We can observe that the performance of all the methods worsens as $P^{(UDP)}$ increases. However, the proposed methods perform always better or equal than the rest of techniques. For $P^{(UDP)} > 0.4$, the proposed methods perform even better than the technique with knowledge of the DDP/UDP condition of the beacons. Similar results were observed for different numbers of beacons, except for $N = 3$, when all the methods are equivalent to the classical LS method.

![Fig. 2: RMSE of the positioning error as a function of $P^{(UDP)}$.](image-url)

$N = 6$ and $\tau_1 = -0.5$ m.
The threshold $\tau_1$ is an important design parameter. A negative value of it closer to zero leads to more discarded combinations. This results in a lower computational complexity for the proposed NEMIRC methods. This is illustrated in Fig. 3, where for the various methods we show the mean number of combinations with computed intermediate estimates. The Rwgh method in [18] was also included for comparison (although it does not depend on $\tau_1$) because it sets an upper bound corresponding to computing estimates for all the possible combinations. The lower bound of the NEMIRC I method and the benchmark for NEMIRC II in Eq. (11) are also displayed. The figure shows the savings in computational complexity of our methods.

We were also interested in how sensitive was the performance of the proposed methods with respect to the value of $\tau_1$. Fig. 4 shows the performances of our methods as functions of $\tau_1$. As can be seen, they are almost unchanged for a wide range of threshold values. The interpretation is that these methods do not necessarily require a very high number of combinations to perform well. Once a sufficient number of well selected combinations is available, adding more combinations does not lead to improved performance. The methods whose performance does not depend on $\tau_1$ were included for comparison purposes.

![Fig. 3: Mean number of combinations for which an intermediate estimate is computed, as a function of $\tau_1$. $N = 6$ and $P^{UDP} = 0.4$.](image1)

![Fig. 4: RMSE of the positioning error as a function of $\tau_1$. $N = 6$ and $P^{UDP} = 0.4$.](image2)

For different values of $N$ and $P^{UDP}$, we obtain similar graphs as in Figures 3 and 4. The mean number of tested combinations for the NEMIRC II is sometimes above and sometimes below the benchmark, usually achieving an improvement with respect to NEMIRC I. The extreme case $N = 3$ is always an exception because a single combination is possible for all the considered techniques.

**B. Methods with knowledge of the DDP range error model**

The RMSE performance of different methods that use knowledge of the range error model for the DDP case as a function of $P^{UDP}$ and with $N = 6$ and $\zeta = 0.3$, is shown in Fig. 5. Several general observations can be made. First, when $P^{UDP} \to 0$, all the methods tend to the same RMSE, because all the beacons are DDP. Then, for the method in [9] a decrease of RMSE can be observed when $P^{UDP}$ tends both to 0 and 1, because in both cases this technique is closer to a pure ML approach due to its use of the UDP error model. On the contrary, for the rest of the methods, the RMSE increases monotonically with $P^{UDP}$. The proposed methods achieve an improvement in RMSE with respect to the na"ive method and towards the benchmarks. All these conclusions can be extended for other values of $N$.

![Fig. 5: RMSE of the positioning error as a function of $P^{UDP}$. $N = 6$ and $\zeta = 0.3$.](image3)

Next we show the effect of $\zeta$ on the proposed methods. In Fig. 6 we see the RMSEs and in Fig. 7 the mean number of computed estimates as functions of $\zeta$, respectively. The PREMI method exhibits a very stable RMSE for almost the whole range of thresholds. This is a convenient feature and its explanation is that once one uses a small number of well selected combinations, the best possible performance has been practically reached. For the extreme case $\zeta = 0$, there is a small increase in the RMSE because all the combinations are used, whereas for the other extreme case $\zeta = 1$, the RMSE is large because all the combinations are rejected except one (the complete set of beacons), and thus, the proposed method boils down to the na"ive ML method. As for the proposed PREMIRC methods, their RMSEs are more sensitive
to $\zeta$, but only for high values of it. To sum up, the PREMI method achieves an improvement with respect to the naïve ML method, reaching the performance of the method in [9]. For the PREMIRC methods, these conclusions hold if the threshold is approximately below 0.5.

In Fig. 7, with the mean number of computed intermediate estimates of the PREMIRC methods, we also see two bounds. The upper bound for both methods is the total number of possible combinations of three or more beacons (the number of combinations used by the Rwgh method, 42 in this case), while the lower bound for PREMIRC I is $\binom{N}{3}$. The benchmark for PREMIRC II is shown too. For the extreme case $\zeta = 0$, no combination is discarded and thus, the number of combinations of the PREMIRC methods are equal to the upper bound. As $\zeta$ increases, more combinations are left out of the computations, and the complexity decreases, reaching PREMIRC I almost its lower bound. For the extreme case $\zeta = 1$, all the combinations of three beacons are discarded and therefore the combination of all the beacons is the only one admitted. Thus, the number of tested combinations is the lower bound of PREMIRC I (combinations of three beacons) plus one. We can also observe that the mean numbers of tested combinations decrease very quickly as the threshold increases from a zero value. This is an appealing feature of the proposed methods, since that implies that the lowest possible complexity can be approached without paying the price of a degraded performance.

We have similar conclusions for $0.2 < P_{UDP} < 0.8$ and $4 \leq N \leq 8$, with the additional observation that the maximum advisable threshold to avoid a rapid increase in RMSE of the PREMIRC methods has a lower value for lower $N$. A threshold around 0.2 or 0.3 allows for good performance in all the cases, that is, for improved RMSE with respect to that of the naïve ML method and close to that of the method with perfect knowledge of the DDP/UDP configurations. At the same time, the mean number of tested combinations for PREMIRC I is near its lower bound, while for PREMIRC II it is sometimes above and sometimes below its benchmark, usually achieving an improvement with respect to PREMIRC I. For these values of $\zeta$, our methods perform at least as well as the method in [9].

VI. CONCLUSIONS

The mitigation of the UDP problem in range-based localization for UWB indoor sensor networks was addressed. We focused on algorithms that exploit as little information as possible so that we minimize their dependence on specific environments and the need for conducting maintenance measurement campaigns. The considered methods were divided into two categories: methods without any knowledge of the range error model, and methods with knowledge of this model, but only for the DDP case. The motivation for the latter is that the model for the DDP measurements is less dependent on the specific environment than that of the UDP measurements. For each of the categories, we proposed an algorithm based on averaging intermediate estimates for different combinations of beacons. Novel criteria for discarding some of these combinations were introduced, all based on inconsistencies revealed by the measurements left out of the combinations. We also proposed algorithms similar to these but with reduced complexity at the cost of some performance loss.

We investigated the performance of the proposed methods with computer simulations. For comparison of their performance, we used methods that have additional information, even though its availability in practice is unrealistic or is difficult to obtain. Extensive simulations show that the proposed methods achieve a performance gain with respect to the methods that use the same amount of information and that they are almost as good as the methods that exploit more information.

REFERENCES

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