Analog Joint Source-Channel Coding for OFDM Systems

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Abstract—Recently, analog Joint Source-Channel Coding (JSCC) has been shown to approach the optimal distortion-cost trade-off when transmitting over AWGN channels. In this work we consider analog JSCC over frequency-selective channels using Orthogonal Frequency Division Multiplexing (OFDM) modulation. Due to its high complexity, optimal MMSE analog JSCC decoding is infeasible in OFDM, hence a practical two-stage decoding approach made up of a MMSE estimator followed by a Maximum Likelihood (ML) decoder is proposed. Three different alternatives for system optimization are considered: non-adaptive coding, adaptive coding, and adaptive coding with precoding. We show that the three analog JSCC transmission strategies approach the optimal distortion-cost trade-off although much better performance is obtained with the adaptive coding with precoding method, specially in Multiple Input Multiple Output (MIMO) OFDM systems.

I. INTRODUCTION

Source compression and channel coding are typically performed separately in most digital communication systems. This strategy, known as the “separation principle”, has been shown to be optimum for both lossless compression [1] and lossy compression of analog sources [2]. However, when digital communication systems are designed to perform close to the optimal distortion-cost trade-off, sources have to be compressed using powerful Vector Quantization (VQ) and entropy coding methods. In addition, data has to be transmitted using capacity approaching digital codes that use long block lengths and introduce significant delay and high computational complexity. Moreover, full redesign of the digital system is required whenever we want to change either the data rate or the distortion target.

Recently, discrete-time analog communication systems based on the transmission of continuous amplitude channel symbols have been proposed as an alternative to digital communication systems. As shown in [3]–[5], using appropriate analog Joint Source Channel Coding (JSCC) techniques, it is possible to approach the optimal distortion-cost trade-off at extremely high data rates with very low complexity and an almost negligible delay.

Most previous work in analog JSCC focuses on AWGN channels. An exception is [6] where the implementation on a Software Defined Radio testbed of a wireless system based on analog JSCC is presented. Excellent performance over wireless channels is attained when the encoder parameters are continuously adapted to the time-varying Channel Signal to Noise Ratio (CSNR). Another exception is [7] where the transmission of analog samples over Multiple Input Multiple Output (MIMO) fading channels is considered.

In this work we consider analog JSCC over frequency-selective channels using OFDM modulation. When combined with MIMO transmission over multiple transmit and receive antennas, the resulting signaling method is referred to as MIMO-OFDM. MIMO-OFDM is the transmission method adopted by the last generation of broadband wireless communication systems due to its ability to achieve large spectral efficiencies while enabling low-complexity equalization of frequency-selective channels.

This paper is organized as follows. Section II reviews the basics of analog JSCC systems while Section III focuses on analog JSCC in MIMO-OFDM systems. Section IV explores the adaptation of the encoder parameters to improve performance. Further improvements can be achieved by linear precoding as explained in Section V. Section VI presents the results of computer experiments and Section VII is devoted to the conclusions.

II. ANALOG JOINT SOURCE-CHANNEL CODING

Figure 1 plots the block diagram of a discrete-time N:1 bandwidth compression analog JSCC transmission system over an AWGN channel. At the transmitter of such a system, N independent and identically-distributed (i.i.d.) source symbols are packed into the source vector $\mathbf{x} = (x_1, x_2, ..., x_N)$ and compressed into one channel symbol $s$. The compression consists of three steps: mapping, $M_\delta(\cdot)$, non-linear transformation, $T_\alpha(\cdot)$, and normalization, $1/\sqrt{\gamma}$. Recent work in analog JSCC [3]–[5], [8] proposes the use of analog mappings based on geometric curves. An example
is the Archimedes spiral [9] whose mathematical expression for \( N = 2 \) is given by

\[
z_\delta(\theta) = \left( \text{sign}(\delta) \frac{\delta}{\pi} \sin \theta, \frac{\delta}{\pi} \cos \theta \right), \quad (1)
\]

where \( \delta \) is the distance between the two neighboring spiral arms and \( \theta \) is the angle from the origin to the point \( z = (z_1, z_2) \) on the curve. Given a specific spiral defined by its \( \delta \) value, the compression function \( M_\delta(\cdot) \) calculates the value \( \hat{\theta} \) corresponding to the point on the spiral that minimizes the distance to \( x \), i.e.

\[
\hat{\theta} = M_\delta(x) = \arg \min_{\theta} ||x - z_\delta(\theta)||^2. \quad (2)
\]

It is important to use the optimum encoding value \( \delta \) in order to achieve the best performance. This parameter determines how the bi-dimensional space is filled up and, therefore, how protected the source symbols are against the noise.

After the mapping, the nonlinear invertible function \( T_\alpha(\hat{\theta}) = \hat{\theta}^\alpha \), denoted the stretching function in [5], is used to transform the channel symbols prior to transmission. Although in most of the literature [3]–[5] \( \alpha = 2 \) is used, the system performance can be significantly improved if \( \alpha \) is optimized together with \( \delta \) [10]. We have empirically determined, via computer simulations, that using \( \alpha = 1.3 \) provides a good overall performance for the case of 2:1 compression in AWGN channels and a wide range of CSNR and \( \delta \) values. Since the analytical optimization of \( \delta \) is not feasible when \( \alpha \neq 2 \) [4], we numerically determined the optimum values of \( \delta \) and store them in a lookup table [11].

Finally, the coded value is normalized by \( \sqrt{\gamma} \) to ensure the average transmitted power is equal to one. Hence, the symbol sent over the channel is given by

\[
s = \frac{T_\alpha(M_\delta(x))}{\sqrt{\gamma}}. \quad (3)
\]

When transmitting over an AWGN channel, the received symbol is given by \( y = s + n \) where \( n \sim \mathcal{N}(0, N_0) \) is a zero-mean Gaussian random variable that represents the channel noise. At reception, the transmitted analog source symbols are decoded from the observation \( y \). Although sub-optimum in analog JSCC, Maximum Likelihood (ML) decoding has been studied in [3], [8] and shown to provide a satisfactory performance over AWGN channels at medium and high CSNR values. The ML estimate \( \hat{x}_{ml} \) is the tuple \((\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N)\) that belongs to the non-linear curve and maximizes the likelihood function \( p(y|x) \), i.e.

\[
\hat{x}_{ml} = \arg \max_{x \in \text{curve}} p(y|x) = \{x | x \in \text{curve and } T_\alpha(M_\delta(x))/\sqrt{\gamma} = y\}. \quad (4)
\]

As shown in [3], [8], ML decoding is equivalent to first applying the inverse function \( T_\alpha^{-1} \) to the observation \( y \) after de-normalization to find an estimate \( \hat{\theta} \) of the transmitted angle \( \theta \), i.e.

\[
\hat{\theta} = T_\alpha^{-1}(\sqrt{\gamma}y) = \text{sign}(y)|\sqrt{\gamma}y|^{-\alpha}. \quad (5)
\]

![Fig. 2. Block diagram of an analog JSCC MIMO-OFDM system.](image)

and then obtaining

\[
\hat{x}_{ml} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N) = z_\Delta(\hat{\theta}). \quad (6)
\]

Notice that the overall ML decoder complexity is extremely low, since the two decoding steps previously described only involve simple mathematical operations.

### III. ANALOG JSCC IN MIMO-OFDM

Figure 2 plots the block diagram of a MIMO-OFDM system with analog JSCC. Discrete-time continuous-amplitude symbols are transmitted over a frequency-selective MIMO channel with \( n_T \) transmit antennas and \( n_R \) receive antennas, using OFDM modulation with \( K \) subcarriers.

Source symbols are first spatially multiplexed over the \( n_T \) transmit antennas. At each transmit antenna, a set of \( KN \) analog source symbols is encoded into \( K \) channel symbols using the \( N:1 \) analog encoding method explained in Section II. Then, each block of \( K \) channel symbols is normalized and transformed into an OFDM symbol. The OFDM symbol \( s_i = \{s_{i,1}, \ldots, s_{i,K}\} \) is then transmitted through antenna \( i \).

We assume a block-fading channel that remains unchanged during the transmission of one OFDM symbol but may vary from one OFDM symbol to another. In the time-domain, the block-fading MIMO channel is represented by the sequence of \( n_R \times n_T \) matrices \( H[l] \) for \( l = 0, \ldots, L - 1 \), being \( L \) the size of the channel memory. In a Rayleigh fading MIMO channel, the entries to \( H[l] \) are complex-valued zero-mean circularly-symmetric Gaussian i.i.d. random variables. In the frequency-domain, the channel response matrix can be expressed as [12]

\[
H_k = \sum_{l=0}^{L-1} R[l]^{1/2} H[l] T[l]^{1/2} \exp \left( -j2\pi lk \right)/K, \quad (7)
\]

where \( H_k \) is the frequency-domain \( n_R \times n_T \) channel matrix response at the \( k \)-th subcarrier \( (k = 1, \ldots, K) \) in the OFDM symbol. Matrices \( R[l] \) and \( T[l] \) are the receive and transmit spatial-correlation matrices, respectively.

Thus, if an OFDM symbol is transmitted from each antenna over the MIMO channel, the received observations \( y_k \) at subcarrier \( k \) are given by

\[
y_k = H_k s_k + n_k, \quad k = 1, \ldots, K \quad (8)
\]

where \( s_k \) is the length \( n_T \) channel symbol vector corresponding to that subcarrier and \( n_k \) is an i.i.d. circularly symmetric complex Gaussian random vector that represents the additive spatially white channel noise.

In analog JSCC, MMSE estimation is the optimal decoding strategy. When considering a MIMO-OFDM system, optimal
decoding consists in the calculation, at each subcarrier \( k \), of the MMSE estimate of the \( N_{T} \) transmitted source symbols \( x_k \) from the received symbol vector \( y_k \), i.e.

\[
\hat{x}_{k,\text{MMSE}} = \mathbb{E}[x_k|y_k] = \int x_k p(x_k|y_k) \, dx_k
\]

\[
= \frac{1}{p(y_k)} \int x_k p(y_k|x_k)p(x_k) \, dx_k. \tag{9}
\]

Since the conditional probability, \( p(y_k|x_k) \), involves the mapping function \( M_s(\cdot) \) which is discontinuous and highly non-linear, the integral in (9) can only be calculated numerically. This implies that the discretization of the set of all possible source values, \( x_k \), is needed. If \( L \) discrete-points are selected per source dimension, we would have to calculate \( L^{N_{T}} \) values for \( p(y_k|x_k) \) and \( p(x_k) \), and then compute the integral in (9). This is infeasible in MIMO-OFDM even for a small number of transmit antennas and subcarriers.

Alternatively, analog JSCC decoding in MIMO-OFDM can be done using a two-stage receiver similar to the one considered in [11]. This receiving strategy involves a first stage where the received symbols are filtered with the aim of minimizing the MSE between the transmitted and estimated symbols, and a second step where ML decoding is applied to the filtered symbols to obtain an estimate of the transmitted source symbols. As shown in [11], the performance of this decoding strategy approaches closely that of the optimal MMSE decoder in AWGN and Rayleigh fading channels while having a complexity similar to that of ML decoding.

Assuming perfect Channel State Information (CSI) available at the receiver, the linear filter \( W_k \) that minimizes the Mean Squared Error (MSE) between the channel symbol vector \( s_k \) and the estimated symbol vector \( \hat{s}_k = W_k y_k \) is given by

\[
W_k = \left( H_k^H H_k + N_0 N_{T} I_{n_T} \right)^{-1} H_k^H, \tag{10}
\]

where the super-index \( H \) represents conjugate transposition. Thus, the set of estimated symbols \( \hat{s}_k \) can be demodulated and used by the ML decoder given by (5) and (6) to calculate an estimate \( \hat{x} \) of the source symbols.

IV. ADAPTIVE ANALOG JSCC

As explained in Section II, better performance is obtained when using the optimal values for \( \delta \) which, at the same time, depend on the CSNR. If no information about the channel is available at the transmitter, it is sensible to use the same \( \delta \), corresponding to the average expected CSNR, to encode all the analog source symbols. This may be adequate in frequency-flat and quasi-static channels where the CSNR remains approximately the same at all subcarriers during a long time but can lead to serious degradation in practical MIMO-OFDM channels where each subcarrier is expected to have a different time-varying CSNR.

Better performance is obtained following an adaptive coding strategy. If we assume that our system is equipped with a feedback channel that regularly sends the CSNR values corresponding to the \( i \)-th transmit antenna and the \( k \)-th subcarrier, the encoding parameter \( \delta \) can be continuously adapted to the channel time variations.

In order to calculate CSNR\(_{i,k}\) we must take into account the detector. When MMSE detection is considered, the filter \( W_k \) does not completely cancel the spatial interference of the MIMO channel. If we consider the residual spatial interference as Gaussian noise that adds to the thermal noise, it can be shown that the equivalent output CSNR can be expressed as

\[
\text{CSNR}_{i,k} = \frac{\mu_{i,k}}{\mu_{i,k} - 1}, \tag{11}
\]

where \( \mu_{i,k} = (W_k H_k)_{ii} \). Thus, the MMSE detector transforms a MIMO-OFDM channel into a set of \( n_T \) SISO-OFDM parallel channels, each one with an equivalent CSNR per subcarrier given by (11).

An important issue regarding system optimization is power normalization. At a first glance, it seems that all subcarriers in the OFDM symbol transmitted over the \( i \)-th antenna should be normalized using the same factor \( \gamma_i \) per antenna. However, notice that this would change the effective CSNR\(_{i,k}\) of the equivalent channel. A better idea is to exploit the CSI provided by the feedback channel and use different factors to normalize the subcarriers with different CSNR. In our case, this idea was implemented calculating a factor \( \gamma_{i,z} \) for different CSNR = \( z \) values and employing them to normalize the channel symbols transmitted into subcarriers with CSNR\(_{i,k} \approx z \). This method is only applied when adaptive coding is used. Notice that this normalization factor must be send alongside the rest of the information, so this normalization adds some extra overhead.

V. ANALOG JSCC MIMO-OFDM WITH LINEAR PRECODING

If CSI is available at transmission, further performance improvements can be obtained if channel symbols are precoded prior to its transmission in a MIMO-OFDM system. As in [14] we will follow the MMSE criterion to jointly design a linear precoder and a linear detector suitable for analog JSCC in MIMO-OFDM.

Recall the MIMO-OFDM signal model defined by (9). Let \( P_k \) be the linear precoder and \( W_k \) the linear detector per subcarrier, respectively. The channel symbol estimations obtained at the detector output are given by

\[
\hat{s}_k = W_k (H_k P_k s_k + n_k),
\]

and, thus, the error between the estimated and transmitted symbols per subcarrier is measured as

\[
e_k = \| s_k - \hat{s}_k \| = \| s_k - W_k (H_k P_k s_k + n_k) \| \tag{12}
\]

The MMSE linear precoder and detector are obtained after solving the following constrained optimization problem

\[
\begin{align*}
\arg \min_{P_k, W_k} & \sum_{k=1}^{K} \mathbb{E}[\text{tr}(e_k e_k^H)] \\
\text{subject to} & \sum_{k=1}^{K} \text{tr}(P_k P_k^H) \leq P_{\text{tx}},
\end{align*}
\]

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where $E[\cdot]$ and $\text{tr}(\cdot)$ denote the expectation and trace operators, respectively, and $P_{tx}$ is the total power available at the transmitter.

Substituting the error expression (12) in (13), differentiating with respect to $P_k$ and $W_k$, and using the Karush-Kuhn-Tucker (KKT) conditions, we arrive at the following equations to obtain the optimal $P_k$ and $W_k$ matrices

$$
P_k = (\lambda I + H_k^H W_k H_k)^{-1} (H_k^H W_k),$$  
(14)

$$
W_k = (P_k^H H_k^H) (n_Y N_0 I + H_k P_k H_k^H)^{-1},$$  
(15)

where $N_0$ is the AWGN noise variance and $\lambda \geq 0$ is the Lagrange multiplier that ensures the total transmit power is equal to $P_{tx}$. Since both equations depend on each other we must calculate the precoder and the detector iteratively. We start assuming an initial precoder equal to the identity matrix and, at each iteration, both the detector and the precoder are sequentially updated using (14) and (15). Notice that the Lagrange multiplier $\lambda$ must be recalculated at each iteration using Newton’s method in order to satisfy the transmit power constraint.

VI. Simulation Results

Computer simulations were carried out to assess the performance of the different analog JSCC MIMO-OFDM systems considered in previous sections. Three different configurations were evaluated: non adaptive coding, adaptive coding and adaptive coding with precoding. Two types of channels were considered: spatially white Rayleigh fading channels, in which the time-domain matrix coefficients are i.i.d. complex-valued circularly symmetric zero-mean Gaussian random variables; and MIMO-OFDM fading channels following standard models.

System performance in terms of the Signal to Distortion Ratio (SDR) with respect to the CSNR. The distortion is the Mean Square Error (MSE) between decoded and source analog symbols, i.e.

$$
\text{MSE} = \frac{1}{N} E\{\|x - \hat{x}\|^2\}.
$$  
(16)

The optimal distortion-cost trade-off is the minimum attainable SDR for a given CSNR. In the literature, this theoretical limit is known as the Optimum Performance Theoretically Attainable (OPTA) and is calculated by equating the rate distortion function to the channel capacity [15]. For $N:1$ compression over a generic $n_R \times n_T$ channel matrix $H$, the OPTA is given by

$$
\frac{1}{N} \log \left( \frac{1}{\text{MSE}} \right) = \frac{1}{n_T} E_H \left[ \log \det \left( I_{n_R} + \frac{\text{CSNR}}{n_T} HH^H \right) \right],
$$

where $E_H[\cdot]$ represents expectation with respect to $H$. In a system with a linear precoding matrix $P$ the capacity corresponds to the one of an equivalent MIMO channel $HP$.

We started by considering the case $n_T = n_R = 1$, i.e. SISO-OFDM. Figures 3 and 4 show the performance of analog JSCC with $N = 2$ in a SISO-OFDM system over a Rayleigh fading channel and a channel following the ITU-Pedestrian B model described in [16], respectively. We consider Gaussian sources and $K = 64$ subcarriers in the OFDM modulation. In the simulated Rayleigh fading channel the memory is $L = 10$. Doppler shift is assumed low enough so that the channel remains static during the transmission of an OFDM symbol and equation (7) is valid. In the ITU-Pedestrian B channel model this condition is met for a reasonable large number of subcarriers.

As observed in both figures, the three proposed strategies approach the OPTA in the whole CSNR region. As expected, the worst performance is obtained when no CSI is available at the transmitter (i.e. non-adaptive coding and no precoding) specially when the CSNR is high (2.5 dB below the OPTA). Using adaptive coding or adaptive coding with precoding increases the performance up to only 1 dB from the OPTA. Notice that the complexity of the first approach is lower than that of the adaptive ones, since it is not required a feedback channel to send the CSI to the transmitter.

Figures 5 and 6 show the performance of analog JSCC in MIMO-OFDM with $n_T = n_R = 4$ and $K = 64$ over a spatially white MIMO Rayleigh fading channel and a channel
following the case D Intelligent Multi-element Transmit and Receive Antennas (I-METRA) channel model described in [17], respectively. Again, for the Rayleigh fading channel we assume that the memory is \( L = 10 \).

In MIMO-OFDM, similar conclusions as those from the SISO case can be extracted, although the effect of adaptive coding and precoding is more significant. The performance when the non-adaptive strategy is employed quickly degrades when increasing the CSNR. Also, the distance between the performance curves corresponding to adaptive coding with precoding is larger \((1 - 1.5 \text{ dB})\). Hence, the knowledge of CSI at transmission in MIMO-OFDM makes it possible to protect the transmitted symbols more strongly against the channel distortion. It is interesting to note that the OPTA of a MIMO-OFDM system with precoding is actually lower than that of the system without precoding. This is because the precoder has been designed to minimize the MSE and not to maximize capacity. Nevertheless, notice that the actual performance of the precoded system is significantly better than that of the system without precoding.

VII. CONCLUSIONS

We have studied the analog transmission of discrete-time coded samples in MIMO-OFDM systems. Source symbols are analog JSCC encoded and sent as OFDM symbols over frequency selective MIMO fading channels. As an alternative to optimal MMSE decoding, we proposed a more practical two-stage receiver made up of a MMSE estimator followed by a Maximum Likelihood (ML) decoder. We studied three different alternatives for system optimization: non-adaptive coding, adaptive coding and adaptive coding with precoding. Simulation results show that the three analog JSCC transmission strategies approach the OPTA although much better performance is obtained with the adaptive coding with precoding method, specially as the number of antennas increases.

REFERENCES