LINK SCHEDULING IN SENSOR NETWORKS FOR ASYMMETRIC AVERAGE CONSENSUS

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ABSTRACT

Wireless Sensor Networks constitute a recent technology where the nodes cooperate to obtain, in a totally distributed way, certain function of the sensed data. One example is the average consensus algorithm, which allows every node to converge to the global average. However, this algorithm presents two major drawbacks in practice. The first one is that instantaneous symmetric links are required, which are hard to ensure in practice because of the presence of wireless interferences. The second one is that all the nodes are required to communicate with all of their local neighbors in every iteration, which can lead to an unbounded delay. In order to solve these issues, we propose a novel link scheduling protocol that activates certain suitable links in each iteration, leading to a new scheme of communications where the links are asymmetric and the communications are performed in an asynchronous manner. This new scheme only requires connectivity and symmetric links on average to guarantee convergence, which are ensured by our link scheduling protocol.

1. INTRODUCTION

Link scheduling protocols have been widely studied in the field of Wireless Sensor Networks (WSNs). These protocols are an efficient mechanism to deal with interferences in which has been possible to include extra features apart from the scheduling itself, such as power control [1], routing [2] or even several at the same time [3]. However, only few works [9] focus on designing a link scheduling protocol to ensure and improve the convergence of an in-network process such as the average consensus protocol. In a consensus protocol, ensuring convergence requires symmetric links and every node must communicate with all of its neighbors at every iteration [4]. The symmetric communications imply the use of an ack-based mechanism, which requires extra communications and time. Moreover, communicating with all the neighboring nodes implies that before starting a new consensus iteration, the nodes have to wait for every other node, which leads to an extra delay in the convergence of the algorithm. A significant variation of the consensus algorithm is the so-called gossip algorithm [5], where each node randomly picks up a neighbor and iteratively computes a symmetric pairwise average with it. However, the number of simultaneous communications and the required pairwise symmetry are still constrained by the interferences. Therefore, a more suitable scenario is to apply the consensus algorithm over an alternative asynchronous and asymmetric scheme of communications, in which we ensure convergence by the simultaneous execution of a new link scheduling protocol. In this scenario, the connectivity of the network is not required to be ensured in each iteration and no node has to wait for any other node, accelerating the consensus process as a consequence. It has been shown in [6][7] that ensuring convergence in this alternative scheme of communications is possible if two alternative conditions are guaranteed: 1) the sequence of connectivity patterns across the iterations provide, on average, a multi-hop path between every pair of nodes and 2) the probability of communicating any pair of nodes is symmetric. This last property presents multiple advantages, among which is the existence of an expression to upper-bound the consensus convergence error [7].

In this paper, the consensus is performed under a new asymmetric and asynchronous scheme of communications in which the convergence of the average consensus algorithm is ensured by the simultaneous execution of a new link scheduling protocol. We explain how our link scheduling protocol provides in this scheme a sequence of connectivity patterns that ensure connectivity of the network on average and symmetric probabilities of connection. We motivate the requirement of having symmetric probabilities of connection and why this requisite can not be satisfied by a CSMA protocol.

The remainder of this paper is structured as follows: The problem formulation and its motivation are given in Section II. The interference model and our link scheduling approach are presented in Section III. We then present, in Section IV, some numerical results to show the efficiency of our approach. Finally, the conclusions are summarized in Section V.
2. PROBLEM FORMULATION

The sequence of instantaneous connectivity patterns that results from applying a link scheduling protocol can be modeled as a time-varying graph \( G(k) = (V, E(k)) \), consisting of a set \( V \) of nodes and a set \( E(k) \subset E \). Note that in a realization of the link scheduling protocol, the subset of links \( E(k) \) does not ensure instantaneous connectivity. Finally, we denote a directed edge from node \( i \) to node \( j \) as \( e_{ij} \), where the presence of an edge \( e_{ij} \) between two nodes indicates that there exists a directed information flow between them. Note that the presence of a link is i.i.d. from iteration to iteration.

Given a time-varying graph \( G(k) \), we can assign an \( N \times N \) adjacency matrix \( A(k) \) where an entry is equal to 1 if \( e_{ij} \in E(k) \) and 0 otherwise. The set of neighbors of a node \( i \) is defined as \( \Omega_i(k) = \{ j : e_{ij} \in E(k) \} \) and the degree matrix \( D(k) \) is a diagonal matrix whose entries are given by \( d_i(k) = |\Omega_i(k)| \). Then, the Laplacian of a graph \( G(k) \) is a matrix defined as \( L(k) = D(k) - A(k) \) whose smallest eigenvalue can be shown to be equal to zero. Then, the matrix \( A(k) \) for a network is non symmetric random and its expected value is \( \bar{A} = P \), where \( P_{ij} \) denotes the probability of establishing a link from node \( j \) to node \( i \).

We consider the general linear update of the state of each sensor \( i \) at time \( k \), using only local data exchange, namely:

\[
x_i(k+1) = W_{ii}(k)x_i(k) + \sum_{j \in \Omega_i(k)} W_{ij}(k)x_j(k)
\]

Since the topology changes with time, we take \( W(k) \) as:

\[
W(k) = I - \alpha L(k)
\]

where \( \alpha \) is a constant independent of time and \( I \) denotes the identity matrix. Since the matrices \( A(k) \) are assumed to be random and independent of each other, the weight matrices \( W(k) \) at the various iteration steps are random, independent of each other, non-symmetric and row-stochastic with its largest eigenvalue equal to one. Moreover, it can be easily shown that \( W(k)1 = 1 \), where \( 1 \) denotes a column vector with all of its entries equal to 1. This property ensures the convergence of the whole network to a common value. However, in order to ensure that this common value corresponds to the average of the initial data, the condition \( 1^T W(k) = 1^T \) must be also satisfied [6]. This condition is not trivially satisfied for asymmetric time-varying topologies without using global knowledge. In the case of asymmetric links, we treat the problem as in [6], where the convergence in the MSE sense is introduced.

Let us assume that the sensor nodes have some initial data at time slot \( k = 0 \). We collect them in a vector, which we call the initial state vector \( x(0) = [x_1(0), x_2(0), \ldots, x_N(0)]^T \), modeled as independent identically distributed random variables with mean \( \mu \) and variance \( \sigma^2 \). Thus the average of the initial state \( x(0) \) is \( x_{\text{avg}} = \frac{1}{N} 1^T E[x(0)] 1 = \mu 1 \), where \( E[] \) denotes the expected value.

Asymmetric topologies are useful to model real networks, where the existence of a link in one iteration depends on time-varying factors such as interferences. The convergence of consensus algorithms having random asymmetric connectivity patterns and assuming that the data is modeled as iid random variables, requires a probabilistic analysis under the MSE paradigm. Thus, we express the convergence of the algorithm in terms of the MSE of the state, defined as:

\[
\text{MSE}(x(k)) = \frac{1}{N} \mathbb{E} \left[ ||x(k) - x_{\text{avg}}||^2 \right]
\]

2.1. Motivation of our approach

In [7], it has been shown that having a symmetric matrix \( P \) is a sufficient condition to ensure convergence in expectation and it also allows to upper bound the MSE. However, it is not easy to satisfy this condition due to the existence of wireless interferences, which produce collisions in the communications. There exist different protocols to avoid these collisions, but each of them lead to different degrees of symmetry in the communications. In order to analyze this property, we define \( \psi \) as a measure of how asymmetric the matrix \( P \) is. We normalize this parameter in such a way that \( 0 \leq \psi \leq 1 \), where \( \psi = 1 \) implies a totally asymmetric connectivity and \( \psi = 0 \) implies a matrix \( P \) totally symmetric. In particular, we define the amount of asymmetry \( \psi \) of the matrix \( P \) as:

\[
\psi = \frac{\sum_{i,j \in V} |P_{ij} - P_{ji}|}{N(N-1)/2}
\]

where we divide by \( N(N-1)/2 \) because this is the case of having maximum asymmetry, that is, if \( P_{ij} > 1 \), then \( P_{ji} = 0 \) and vice versa. Figure 1 shows that the MSE increases as a function of the asymmetry of the network, which is represented by the parameter \( \psi \). In this paper, we provide a link scheduling protocol that guarantees a symmetric matrix \( P \), ensuring convergence and reducing the MSE.

![Fig. 1. Evolution of the MSE as a function of the value of \( \psi \).](image-url)
3. LINK SCHEDULING PROTOCOL

In this section, we propose a new link scheduling protocol that ensures the convergence of the average consensus algorithm with certain degree of accuracy. This protocol provides symmetric probabilities of communication, which can not be guaranteed by a CSMA protocol, as discussed later. Moreover, our protocol does not require using global knowledge, such as the distance between the network nodes, which are required by most of the existing approaches in the literature, making them incompatible with the consensus algorithm.

3.1. Interference model

We consider a WSN with $N$ nodes, where each of them is equipped with an omni-directional antenna. A node having this hardware cannot transmit and receive at the same time and can only communicate over one link at a time. The transmit power of a node is denoted by $P_i$ and the channel gain between a transmitter node $j$ and a receiving node $i$ is $\frac{1}{r_{ij}}$, where $\gamma \geq 2$ is the path loss exponent.

We assume the SINR interference model according to which the successful reception of a packet sent by node $j$ and destined to node $i$ depends on the SINR at node $i$, that is, a packet between $j$ and $i$ is correctly received only if

$$\frac{P_j}{r_{ij}} \varsigma + \sum_{v \in V, v \neq j} \frac{P_v}{r_{iv}} \geq \beta$$  \hfill (5)

where $\varsigma$ is the background noise and $\beta$ is a constant threshold. In this model, all the simultaneous transmissions are considered when evaluating whether a single transmission is valid. Therefore, every link can affect each other even if they do not share the intended receiver, which makes the problem to be NP-hard [8]. This implies that an heuristic link scheduling protocol should be used.

3.2. Link scheduling protocol

We propose a low complexity link scheduling protocol that allows a consensus process to be executed in a totally distributed fashion, avoiding collisions and ensuring its convergence. In order to explain the proposed method, we define three different sets to classify the links during each step, denoted by $n$, of the link scheduling protocol:

- **ACTIVATED $A(n)$**: every link that is chosen to be activated in each step of the link scheduling protocol is added to this set.
- **INHIBITED $I(n)$**: every link that is inhibited by an active link is added to this set.
- **UNCLASSIFIED $U(n)$**: every link that does not belong to any of the previous sets, belongs to this set.

Note that the network nodes do not need to know these three sets of links. In practice, a node only requires to know and update the state of the links within its local inhibition area. Therefore, taking into account (5), the maximum transmission radius denoted $R_{\text{max}}$, is defined as the maximum distance up to which a packet can be correctly received in absence of interference:

$$R_{\text{max}} = \left( \frac{P_i}{\beta \varsigma \beta} \right)^{\frac{1}{\gamma}}$$  \hfill (6)

Since a transmission from a node located at a distance equal to $R_{\text{max}}$ implies that no other link can be scheduled simultaneously without collision, we assume that the length of the links to be scheduled, in each iteration of the consensus process, is lower or equal than the product of $R_{\text{max}}$ and a constant factor $0 < \rho < 1$. Although, the value of $\rho$ must be large enough $\rho R_{\text{max}} = \sqrt{2 \log N} \sqrt{N}$ to ensure connectivity with high probability [5], it is accomplished that, when smaller, the greater number of simultaneous links can be obtained in each step of the link scheduling protocol.

Therefore, our method consists in randomly picking links from the set $U$, and define an associated inhibition area that contains all the links that are included in $I$, when that link is chosen. The inhibition radius $R_{\text{inh}}$ is defined from (6), and it refers to the distance from which all the nodes $j$, that are at distance $r_{ij} \leq R_{\text{inh}}$ from the receiving node $i$, are inhibited. In a more formal way, we have the following expression:

$$R_{\text{inh}} \geq \left( \frac{\beta \eta P_i}{P_i (\rho R_{\text{max}})^{2} - \varsigma \beta} \right)^{\frac{1}{\gamma}}$$  \hfill (7)

where $\eta$ is an estimation of the maximum number of simultaneous transmissions in the current deployment. Finally, substituting in (7) the expression of (6) becomes:

$$R_{\text{inh}} \geq \left( \frac{\beta \eta P_i}{\varsigma \beta} \left( \frac{1}{\rho} - \varsigma \beta \right) \right)^{\frac{1}{\gamma}}$$  \hfill (8)

![Fig. 2. Relation between the different areas and radii that are used by our link scheduling protocol.](image-url)
This equation is obtained assuming that every transmitter is located exactly at distance $\rho R_{\text{max}}$ of the corresponding receiver in order to ensure an inhibition area large enough to allow $\eta$ simultaneous transmission without collisions. Fig. 2 shows the relation between the different areas and radii. Since the parameter $\eta$ has to be estimated a priori from the deployment, an inherent error can occur, which implies the following scenarios: 1) if the estimated value $\hat{\eta}$ is larger than the optimal one $\eta_{\text{opt}}$, the radius of the inhibition area becomes larger and the final number of simultaneous links is reduced, that is, $\eta \leq \eta_{\text{opt}} \leq \hat{\eta}$ and 2) if the estimated value $\hat{\eta}$ is smaller than the optimal one $\eta_{\text{opt}}$, we assume that the protocol is finalized when $\hat{\eta}$ simultaneous links are obtained, that is, $\eta = \hat{\eta} \leq \eta_{\text{opt}}$.

Algorithm 1 LINK SCHEDULING

Require: $\mathcal{A}, \mathcal{I}, \mathcal{U}, \hat{\eta}$
Ensure: $\mathcal{U}$ is empty OR $\text{cont} = \hat{\eta}$

cont = 0
while $\mathcal{U}$ is not empty AND $\text{cont} \leq \hat{\eta}$ do
    $e_{ds} = \text{Randomly choose a link from } \mathcal{U}$
    add $e_{ds}$ to $\mathcal{A}$
    remove $e_{ds}$ from $\mathcal{U}$
    for $i = 1$ to $N$ do
        for $j = 1$ to $N$ do
            if $r_{ij} \leq R_{\text{inh}}$ AND $e_{ij} \notin \mathcal{A}$ then
                add $e_{ij}$ to $\mathcal{I}$
                remove $e_{ij}$ from $\mathcal{U}$
            end if
        end for
    end for
    $\text{cont} = \text{cont} + 1$
end while

Algorithm 1 ensures that a connectivity pattern with no collisions is created before every iteration of the consensus process. Note that in practice all the activation and inhibition steps are performed in an asynchronous manner, the only assumption is that two nodes in the same inhibition area do not wake up at the same time. Initially, there are no activated or inhibited links, so that the sets $\mathcal{A}$ and $\mathcal{I}$ are empty, while the set $\mathcal{U}$ contains every link in $\mathcal{E}$. A link between a transmission node $s$ and a receiving node $d$ is randomly chosen from the set $\mathcal{U}$, e.g., a random timer of a node expires, the node wakes up and randomly chooses a neighbor to communicate with. As a consequence, this link is added to the set of active links $\mathcal{A}$. Then, the activation of this new link implies that no other transmission is made simultaneously around the receiving node $d$. Therefore, the links between any pair of nodes $i$ and $j$ having $r_{ij} \leq R_{\text{inh}}$ are added to the set $\mathcal{I}$ and removed from the set $\mathcal{U}$. The resulting sequence of connectivity patterns does not ensure connectivity at one iteration of the consensus algorithm, but since the patterns are generated randomly over the iterations of the consensus process, this allows to create a totally connected network on average.

3.3. Ensuring symmetric probabilities of connection

In order to show that our link scheduling protocol is able to ensure a symmetric matrix $\mathbf{P}$, we relate the probability of activating a link with the size of its corresponding inhibition area. In particular, the probability of activating the link $e_{ij}$ in the $n$-th step of the link scheduling $P_{ij}^A(n)$, depends on both the probability of being chosen from the set $\mathcal{U}(n)$ and the probability of remaining in the set $\mathcal{U}(n)$. The former probability depends on the cardinality of set $\mathcal{U}(n)$. Therefore, if we define a random variable $|\mathcal{U}(n)|$ modeling the number of links in the set $\mathcal{U}$, in the activation step $n$, we can express this probability as $\mathbb{E}\left[\frac{1}{|\mathcal{U}(n)|}\right]$. The latter probability, assuming the node $i$ as the endpoint of the link $e_{ij}$, can be defined as the probability of not being activated or inhibited in any of the previous steps. If we define the number of unclassified links in the inhibition area of node $i$ in the $n$-th step, as the set $S_i(n) = \{e_{ij} \in \mathcal{U}(n) : r_{ij} \leq R_{\text{inh}}\}$, the probability of activating the link $e_{ij}$, in the $n$-th step, can be expressed as:

$$P_{ij}^A(n) = \mathbb{E}\left[\frac{1}{|\mathcal{U}(n)|}\right] \left(1 - \sum_{l=1}^{n-1} \mathbb{E}\left[|S_i(l)|\right] + P_{ij}^A(l)\right)$$

and the total probability of activating the link $e_{ij}$ is:

$$P_{ij}^A = \sum_{n=1}^{\eta} P_{ij}^A(n), \forall e_{ij} \in \mathcal{E}$$

where $\eta$ is the number of activation steps during the link scheduling protocol. Note that since only one link is activated in each of the activation steps, $\eta$ links become finally activated. Therefore, if we move the center of every inhibition area to the center of the link, as shown in Fig. 3, we ensure that $|S_i(l)| = |S_j(l)| \forall l = 1, 2, ..., \eta$. This means that on average, the number of links in the inhibition area, for a given directed link and its symmetric counterpart, is the same, enforcing a symmetric matrix $\mathbf{P}$. Finally, the scheme of Fig. 3...
implies that we have to add a constant term $\delta$ to the inhibition radius $R_{inh}$. It is easy to check that for ensuring no collisions, we need to add $\delta \geq \frac{\rho R_{max}}{2}$ to every inhibition radius $R_{inh}$. In the case of a CSMA protocol, an intended transmitter decides weather or not to transmit based on the energy of the channel and a constant given threshold. This implies having inhibition areas centered in the transmitter. Therefore, the number of links in each inhibition area depends on the topology of the network and the degree of each node. Then, $|S_i(l)| \neq |S_j(l)| \forall l = 1, 2, ..., \eta$, in general.

4. SIMULATION RESULTS

We model a WSN as a uniformly randomly deployed network of $N = 100$ nodes inside a $2D$ unit square area, where each node measurement is modeled as an independent Gaussian random variable with mean $\mu = 20$ and variance $\sigma^2_0 = 8$. The information is mixed as described in (2) where the instantaneous topology determines which data is mixed.

Channel gains are computed based on node positions, and on the radio propagation model. Radio signal propagation is assumed to follow log-normal shadowing, with path loss exponent $\gamma = 3$. Additionally, for a given background noise $G = 10^{-9} mW$ and a given value of $\beta = 1$, we choose a combination of the values $P_i$ and $\rho$ to ensure connectivity on average, that is, $\rho R_{max} = \sqrt{2 \log N/N}$, which has been shown in [5] to ensure connectivity with high probability in a random geometric graph.

Fig. 4 shows the convergence behavior of the average consensus algorithm when this is simultaneously executed with our link scheduling and also with a CSMA based protocol. Our protocol provides smaller values of the MSE because it ensures symmetric probabilities of connection.

5. CONCLUSIONS

In this paper, we have considered the problem of the average consensus in WSNs under an accurate interference model in which the correct packet reception at a receiving node depends on the SINR. We propose a decentralized link scheduling that leads to connectivity patterns that are free of collisions. We devise a probabilistic model where the probability of connection between any pair of nodes is symmetric, ensuring convergence with certain degree of accuracy. Moreover, we show experimentally how the average consensus process converges when it is simultaneously executed with both our link scheduling and a CSMA-based protocol.

6. REFERENCES


